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GIMP application

Subject: **Math (II grade)**

Topic: **Area of a regular polygon**

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Topic: Area of a regular polygon

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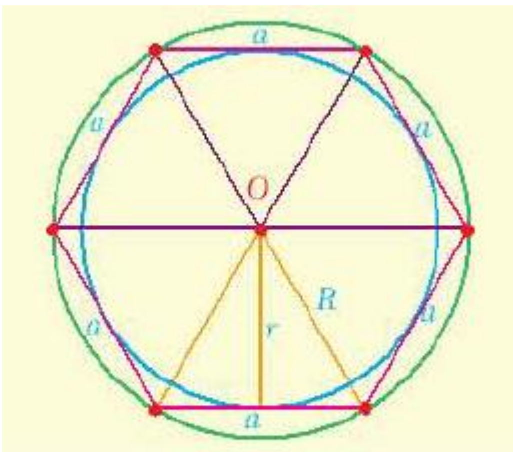
Abstract: Recall that a regular polygon is a polygon which is both equilateral (all side lengths are equal) and equiangular (all angles are congruent).

Area of a regular polygon

Definition: All regular polygons can be drawn inside a circle so that all their vertices lie on the circle. This circle is called the circumscribed circle of the polygon. In the figure opposite, the larger circle is the circumscribed circle of the hexagon.

We can also always draw a circle inside a regular polygon such that the circle is tangent to every side of the polygon. This circle is called the inscribed circle of the polygon. In the figure opposite, the smaller circle is the inscribed circle of the hexagon.

The distance between the center of the regular polygon and the one of its sides is called the apothem of the polygon. In the figure, the length r is the apothem of the hexagon.



Note: In a regular polygon, the centers of the circumscribed and inscribed circles coincide at the same point. This point is called the center of the regular polygon.

Rule:

1. The distance between the center of a polygon and any of its vertices is the same as the radius of its circumscribed circle.

2. The apothem of a regular polygon is equal to the radius of its inscribed circle.

Note: We generally use r to denote the radius of an inscribed circle, and R the radius of a circumscribed circle.

Theorem: The area of a regular polygon is half the product of its apothem r and its perimeter L:

$$P = \frac{1}{2} rL .$$

We can write this alternatively as P=s·r, where s is half perimeter of the polygon.

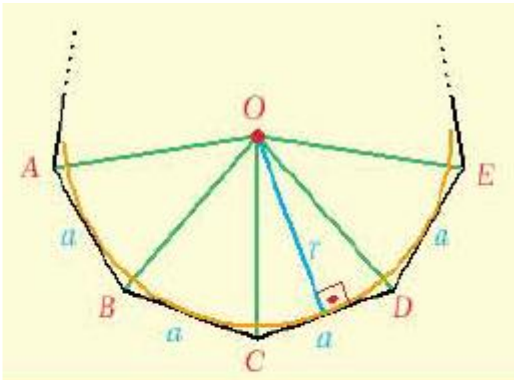
Proof: Look at the figure. As we have seen, the distance between a center of a regular polygon and any vertex is R, and the distance between the center and a side is r.

Notice that if we connect O with vertices A, B C, etc., we obtain a number of congruent triangles. Let this number be n.

The area of each triangle is $\frac{a \cdot h}{2}$.

There are n triangles, so $\Pr.p. = n\frac{a \cdot r}{2}$.

Since the perimeter of the polygon is n·a, the expression for its area reduces to $P = \frac{1}{2} r \cdot L = s \cdot r$, as required.



Note: We know in any regular polygon, the triangles that are created when we connect the center of polygon to the vertices are all congruent. So one of the central angles is $\alpha = \frac{360°}{n}$.

This means that we can also find the area of a regular polygon by using trigonometry: if R is the distance between the center and a vertex of a regular polygon then its area is $n \cdot (\frac{1}{2} \cdot R \cdot R \cdot \sin\alpha)$, i.e. $P = \frac{n \cdot R^2 \cdot \sin\alpha}{2}$.

Example 1: A regular polygon is given. The radius of its inscribed circle is 4cm, one side measures 6cm and its area is 96cm^2. How many sides does this polygon have?

Solution: Let the number of sides be n, then the perimeter L=6n. Using $P=\frac{1}{2}r\cdot L$ gives us $96=\frac{1}{2}\cdot 4\cdot 6n=12n$ i.e. n=8. So the polygon has eight sides.

Example 2: The radius of the circumscribed circle of a regular octagon ic 4cm. Find the area of the octagon.

Solution: n=8 and R=4cm are given, and $\alpha=\frac{360^\circ}{n}=\frac{360^\circ}{8}=45^\circ$.

Using $P=\frac{n\cdot R^2\cdot\sin\alpha}{2}$ gives $P=\frac{8\cdot 4^2\cdot\sin 45^\circ}{2}=64\cdot\frac{\sqrt{2}}{2}=32\sqrt{2}cm^2$.